Lesson 1.1, pp. 11–13

1. **a)** $D = \{x \in \mathbf{R}\}$;
$R = \{y \in \mathbf{R} \mid -4 \le y \le -2\}$; This is a function because it passes the vertical line test.

b) $D = \{x \in \mathbf{R} \mid -1 \le x \le 7\}$;
$R = \{y \in \mathbf{R} \mid -3 \le y \le 1\}$; This is a function because it passes the vertical line test.

c) $D = \{1, 2, 3, 4\}$;
$R = \{-5, 4, 7, 9, 11\}$; This is not a function because 1 is sent to more than one element in the range.

d) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$; This is a function because every element in the domain produces exactly one element in the range.

e) $D = \{-4, -3, 1, 2\}$; $R = \{0, 1, 2, 3\}$; This is a function because every element of the domain is sent to exactly one element in the range.

f) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \le 0\}$; This is a function because every element in the domain produces exactly one element in the range.

2. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \le -3\}$; This is a function because every element in the domain produces exactly one element in the range.

b) $D = \{x \in \mathbf{R} \mid x \ne -3\}$;
$R = \{y \in \mathbf{R} \mid y \ne 0\}$; This is a function because every element in the domain produces exactly one element in the range.

c) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > 0\}$; This is a function because every element in the domain produces exactly one element in the range.

d) $D = \{x \in \mathbf{R}\}$;
$R = \{y \in \mathbf{R} \mid 0 \le y \le 2\}$; This is a function because every element in the domain produces exactly one element in the range.

e) $D = \{x \in \mathbf{R} \mid -3 \le x \le 3\}$;
$R = \{y \in \mathbf{R} \mid -3 \le y \le 3\}$; This is not a function because (0, 3) and (0, 3) are both in the relation.

f) $D = \{x \in \mathbf{R}\}$;
$R = \{y \in \mathbf{R} \mid -2 \le y \le 2\}$; This is a function because every element in the domain produces exactly one element in the range.

3. **a)** function; $D = \{1, 3, 5, 7\}$;
$R = \{2, 4, 6\}$

b) function; $D = \{0, 1, 2, 5\}$;
$R = \{-1, 3, 6\}$

c) function; $D = \{0, 1, 2, 3\}$; $R = \{2, 4\}$

d) not a function; $D = \{2, 6, 8\}$;
$R = \{1, 3, 5, 7\}$

e) not a function; $D = \{1, 10, 100\}$;
$R = \{0, 1, 2, 3\}$

f) function; $D = \{1, 2, 3, 4\}$;
$R = \{1, 2, 3, 4\}$

4. **a)** function; $D = \{x \in \mathbf{R}\}$;
$R = \{y \in \mathbf{R} \mid y \ge 2\}$.

b) not a function; $D = \{x \in \mathbf{R} \mid x \ge 2\}$;
$R = \{y \in \mathbf{R}\}$

c) function; $D = \{x \in \mathbf{R}\}$;
$R = \{y \in \mathbf{R} \mid y \ge -0.5\}$

d) not a function; $D = \{x \in \mathbf{R} \mid x \ge 0\}$;
$R = \{y \in \mathbf{R}\}$

e) function; $D = \{x \in \mathbf{R} \mid x \ne 0\}$;
$R = \{y \in \mathbf{R} \mid y \ne 0\}$

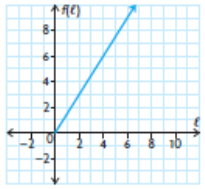
f) function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$

5. **a)** $y = x + 3$ **c)** $y = 3(x - 2)$
b) $y = 2x - 5$ **d)** $y = -x + 5$

6. **a)** The length is twice the width.

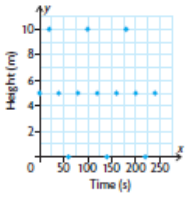
b) $f(l) = \frac{3}{2}l$

c)



d) length = 8 m; width = 4 m

7. **a)**

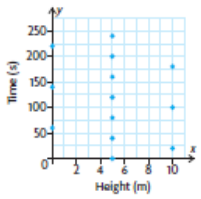


b) $D = \{0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240\}$

c) $R = \{0, 5, 10\}$

d) It is a function because it passes the vertical line test.

e)



f) It is not a function because (5, 0) and (5, 40) are both in the relation.

8. **a)** {(1, 2), (3, 4), (5, 6)}

b) {(1, 2), (3, 2), (5, 6)}

c) {(2, 1), (2, 3), (5, 6)}

9. If a vertical line passes through a function and hits two points, those two points have identical x-coordinates and different y-coordinates. This means that one x-coordinate is sent to two different elements in the range, violating the definition of *function*.

10. **a)** Yes, because the distance from (4, 3) to (0, 0) is 5.

b) No, because the distance from (1, 5) to (0, 0) is not 5.

c) No, because (4, 3) and (4, −3) are both in the relation.

11. **a)** $g(x) = x^2 + 3$

b) $g(3) - g(2) = 12 - 7$
$= 5$
$g(3 - 2) = g(1)$
$= 4$
So, $g(3) - g(2) \ne g(3 - 2)$.

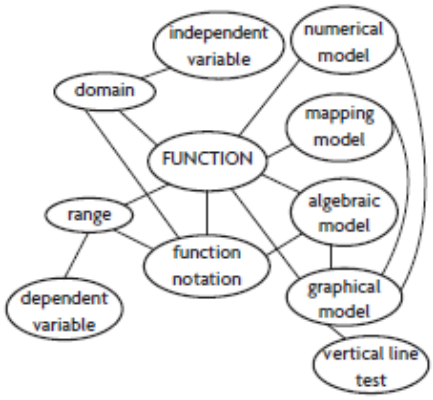
12. **a)** $f(6) = 12$; $f(7) = 8$; $f(8) = 15$

b) Yes, $f(15) = f(3) \times f(5)$

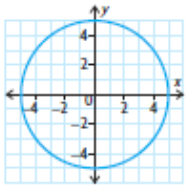
c) Yes, $f(12) = f(3) \times f(4)$

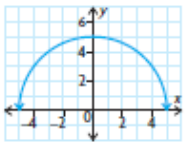
d) Yes, there are others that will work. $f(a) \times f(b) = f(a \times b)$ whenever a and b have no common factors other than 1.

13. Answers may vary. For example:



14.





The first is not a function because it fails the vertical line test:
$D = \{x \in \mathbf{R} \mid -5 \le x \le 5\}$;
$R = \{y \in \mathbf{R} \mid -5 \le y \le 5\}$.
The second is a function because it passes the vertical line test:
$D = \{x \in \mathbf{R} \mid -5 \le x \le 5\}$;
$R = \{y \in \mathbf{R} \mid 0 \le y \le 5\}$.

15. x is a function of y if the graph passes the horizontal line test. This occurs when any horizontal line hits the graph at most once.