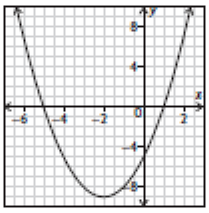
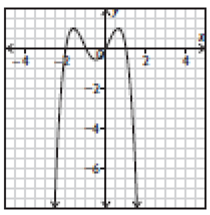
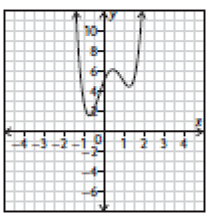
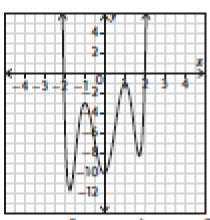
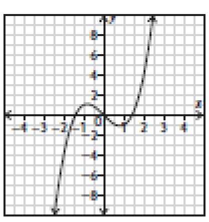
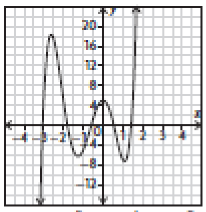
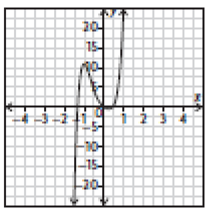
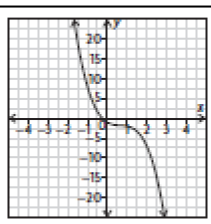
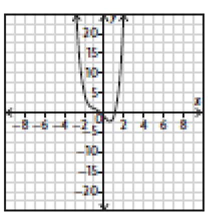
Answers to Investigate the Math

A.

Equation and Graph	Degree	Even or Odd Degree?	Leading Coefficient	End Behaviours		Number of Turning Points
				$x \to -\infty$	$x \to +\infty$	
a)  $f(x) = x^2 + 4x - 5$	2	even	+1	$y \to +\infty$	$y \to +\infty$	1
b)  $f(x) = -x^4 - 2x^3 + x^2 + 2x$	4	even	−1	$y \to -\infty$	$y \to -\infty$	3
c)  $f(x) = 3x^4 - 4x^3 - 4x^2 + 5x + 5$	4	even	3	$y \to +\infty$	$y \to +\infty$	3
d)  $f(x) = 2x^6 - 12x^4 + 18x^2 + x - 10$	6	even	2	$y \to +\infty$	$y \to +\infty$	5
e)  $f(x) = x^3 - 2x$	3	odd	1	$y \to -\infty$	$y \to +\infty$	2

Answers to Investigate the Math (Continued)

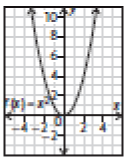
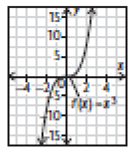
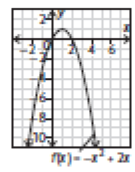
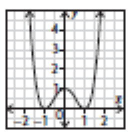
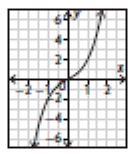
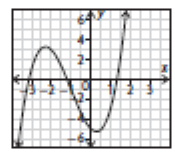
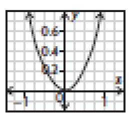
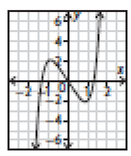
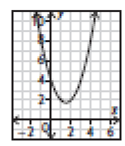
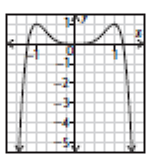
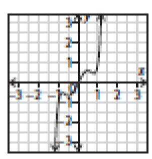
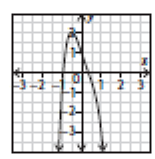
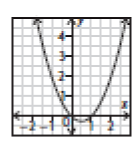
A.

Equation and Graph	Degree	Even or Odd Degree?	Leading Coefficient	End Behaviours $x \to -\infty$	End Behaviours $x \to +\infty$	Number of Turning Points
f)  $f(x) = 2x^5 + 7x^4 - 3x^3 - 18x^2 + 5$	5	odd	2	$y \to -\infty$	$y \to +\infty$	4
g)  $f(x) = 5x^5 + 5x^4 - 2x^3 + 4x^2 - 3x$	5	odd	5	$y \to -\infty$	$y \to +\infty$	2
h)  $f(x) = -2x^3 + 4x^2 - 3x - 1$	3	odd	−2	$y \to +\infty$	$y \to -\infty$	0
i)  $f(x) = x^4 + 2x^3 - 3x - 1$	4	even	1	$y \to +\infty$	$y \to +\infty$	1

B. Answers may vary. Odd degrees have opposite end behaviour; even degrees have identical end behaviour. If the leading coefficient is positive and the degree is even, $y \to +\infty$ in both directions, whereas if the leading coefficient is negative, $y \to -\infty$ in both directions. In an odd degree polynomial, if the leading coefficient is positive, then as $x \to +\infty$, $y \to +\infty$ and as $x \to -\infty$, $y \to -\infty$, and vice versa for a negative leading coefficient. For a polynomial of degree n, there are at most $n - 1$ turning points.

C. Answers may vary, but the new polynomials should still meet the above observations.

D. $n - 1$
E.

Even Functions	Odd Functions	Neither Even nor Odd Functions
(symmetry in the y-axis) $f(x) = x^2$ 	(rotational symmetry around the origin) $f(x) = x^3$ 	(neither of these symmetries) $f(x) = -x^2 + 2x$ 
$f(x) = x^4 - 2x^2 + 1$ 	$f(x) = x^3 + x$ 	$f(x) = x^3 + 3x^2 - 2x - 5$ 
$f(x) = \frac{1}{2}x^{10} - \frac{1}{3}x^4 + x^2$ 	$f(x) = x^5 - 3x$ 	$f(x) = x^2 - 3x + 4$ 
$f(x) = -2x^6 + 3x^4$ 	$f(x) = 2x^7 - 3x^3 + 2x$ 	$f(x) = -3x^4 + 2x^3 - 3x + 1$ 
		$f(x) = x^2 - x$ 

G. No, for example, ix).
H. No, for example, ii).
I. Polynomials of odd degree have opposite end behaviour; polynomials of even degree have identical end behaviour. If the leading coefficient is positive and the degree is even, $y \to +\infty$ in both directions, whereas if the leading coefficient is negative, $y \to -\infty$ in both directions. In an odd degree polynomial, if the leading coefficient is positive, then as $x \to +\infty$, $y \to +\infty$ and as $x \to -\infty$, $y \to -\infty$, and vice versa for a negative leading coefficient. For a polynomial of degree n, there are at most $n - 1$ turning points. To check symmetry, check $f(-x)$.

Answers to Reflecting

J. They have identical end behaviour on either side.
K. Because their end behaviour is opposite, which means the graph must pass through the x-axis at least once.
L. Yes, for example, $f(x) = x^2 + 1$.
M. It is not possible to predict it precisely, but if the degree is n, the number of zeros is at most n, and a polynomial of odd degree must have at least one real zero.