Critical Points. Local Extrema. (4.2)

B1. I can make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching;

"I can determine all critical points and local extrema for any function. Also, I can graph this function. I can apply what I have learned in unfamiliar and familiar settings."

Critical Points

Given $y = f(x)$. At $x = c$ a **critical point** exists if $f'(c) = 0$ or $f'(c)$ is undefined, as long as $x = c$ belongs to the domain of f.

Local Extrema

Collectively, local maximums and local minimums are local extreme points, or **local extrema**.

Recall:

Local Maximum at $x = c$

Given $y = f(x)$. For values of x near c ... A **local maximum** at $x = c$ exists if $f(c) > f(x)$ for these values. Thus, $f'(x) > 0$ for the values $x < c$ and $f'(x) < 0$ for the values $x > c$.

Local Minimum at $x = c$

Given $y = f(x)$. For values of x near c ... A **local minimum** at $x = c$ exists if $f(c) < f(x)$ for these values. Thus, $f'(x) < 0$ for the values $x < c$ and $f'(x) > 0$ for the values $x > c$.

Thus, local extrema are examples of critical numbers. Here are some examples:

But not all critical numbers are local extrema...

Example 1 Find all critical numbers for $y = x^3$

First Derivative Test (for local extrema)

Let c be a critical number of a function f.

When moving through x-values from left to right:

- if $f'(x)$ changes from negative to positive at c, then $(c, f(c))$ is a **local minimum** of f.
- if $f'(x)$ changes from positive to negative at c, then $(c, f(c))$ is a **local maximum** of f.
- if $f'(x)$ does not change its sign at c, then $(c, f(c))$ is neither a local minimum or a local maximum.

Example 2 Given $f(x) = 2(x-4)^{\frac{2}{3}}$. Determine:
a) the domain of f
b) all critical numbers https://www.desmos.com/calculator/v7zhatfwom

Example 3 Given $f(x) = 2(x-4)^{\frac{1}{3}}$. Determine:
a) the domain of f
b) all critical numbers https://www.desmos.com/calculator/k9isijqwzz

Example 4 Given $f(x) = 3x^4 - 8x^3 - 24x^2 + 96x + 12$, determine all critical values. Determine if these critical values give a local maximum, local minimum, or neither.

https://www.desmos.com/calculator/vq6q88bgnc

This work is absolutely critical...Page 178...#3b, 4b, 5d, 7cd, 8, 9, 12, 13, 14d, 15*. *In the solution for 15a)...

Insert ", $d = -9$" after "$c = 0$".