Asymptotes (4.3)

B1. I can make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching;

"I will use limits to find the equations of all asymptotes. I can graph all rational functions. I can apply what I have learned in unfamiliar and familiar settings."

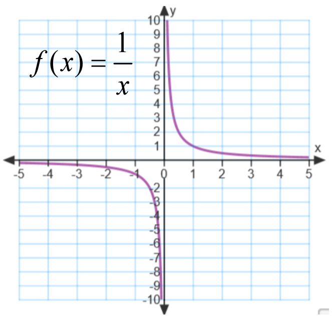
Recall: A **rational function** is of the form: $f(x) = \dfrac{p(x)}{q(x)}$

where $p(x), q(x)$ are polynomial functions ($q(x) \neq 0$).

An **asymptote** is a line (or curve) which is tangent to the function at $\pm\infty$. The distance between the function and the asymptote tend to zero as they move toward $\pm\infty$.

Asymptotes can be vertical, horizontal and oblique.

Example I



Evaluate, if the limit exists:

a) $\lim\limits_{x\to 0^-} \dfrac{1}{x}$

b) $\lim\limits_{x\to 0^+} \dfrac{1}{x}$

c) $\lim\limits_{x\to 0} \dfrac{1}{x}$

d) $\lim\limits_{x\to \infty} \dfrac{1}{x}$

e) $\lim\limits_{x\to -\infty} \dfrac{1}{x}$

Vertical Asymptotes and Infinite Limits

The graph of $f(x)$ has a vertical asymptote, $x = c$, if one of the following infinite limit statements is true:

$$\lim_{x\to c^-} f(x) = +\infty, \quad \lim_{x\to c^-} f(x) = -\infty, \quad \lim_{x\to c^+} f(x) = +\infty \quad \text{or} \quad \lim_{x\to c^+} f(x) = -\infty$$

Vertical Asymptotes of Rational Functions

A rational function of the form $f(x) = \dfrac{p(x)}{q(x)}$ has a vertical asymptote $x = c$ if $q(c) = 0$ and $p(c) \neq 0$.

Horizontal Asymptotes and Limits at Infinity

If $\lim\limits_{x\to +\infty} f(x) = L$ or $\lim\limits_{x\to -\infty} f(x) = L$, we say that the line $y = L$ is a horizontal asymptote of the graph of $f(x)$.

A horizontal asymptote exists when the degree of the numerator is less than or equal to the degree of the denominator.

Example II Determine all vertical and horizontal asymptotes for $f(x) = \dfrac{x^2 - 4x - 5}{x^2 - 7x + 10}$ and describe the behaviour near the asymptotes.

Use desmos to verify your results.

https://www.desmos.com/calculator/qsxxajqrc4

Oblique Asymptote

An asymptote that is neither horizontal nor vertical is an **oblique asymptote**.

An oblique asymptote exists when the degree of the numerator is greater than the degree of the denominator in a rational function.

Graphing

Algorithm for Curve ~~Sketching~~ (so far)

To sketch a curve, apply these steps in the order given.

1. Check for any discontinuities in the domain. Determine if there are vertical asymptotes at these discontinuities, and determine the direction from which the curve approaches these asymptotes.
2. Find **both intercepts**.
3. Find any critical points.
4. Use the first derivative test to determine the type of critical points that may be present.
5. **Test end behaviour** by determining $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.
6. Construct an interval of increase/decrease table and identify all local or absolute extrema.
7. ~~Sketc~~h the curve.
   Graph

Example III Graph: $f(x) = \dfrac{x^2 - 3x + 6}{x - 1}$

Verify with desmos : https://www.desmos.com/calculator/pqckzjltnb

Nothing "sketchy" about these! Page 193...#3abc, 4de, 5cd, 7*, 8, 9bc, 10 Graph adef, 11, 14, 15, 16
*for 7a the answer is y = 3x + 7