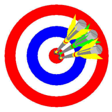
Concavity and Points of Inflection (4.4)

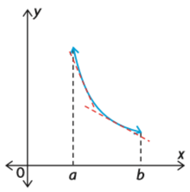
B1. I can make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching;



"I know how to determine if a function is concave up or concave down. I can apply the second derivative test to determine local extrema. I know how to find a point of inflection. I can apply what I have learned in familiar and unfamiliar settings."

Recall: Given: $y = f(x)$. The derivative of y with respect to x is $y' = f'(x)$. It represents

A



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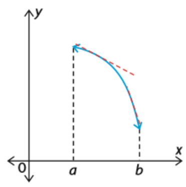

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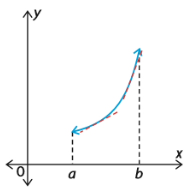

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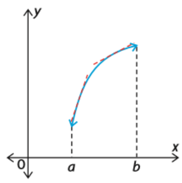


Recall: Given: $y = f(x)$. The ***second*** derivative of y with respect to x is $y'' = f''(x)$. It represents

Which graphs above demonstrate that the slopes of the tangents of $y = f(x)$ are increasing?

Which graphs above demonstrate that the slopes of the tangents of $y = f(x)$ are decreasing?

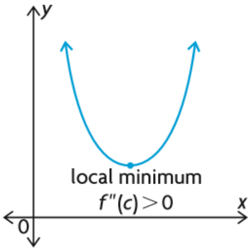
concave up The graph of $y = f(x)$ is **concave up** on an interval $a \le x \le b$ in which the slopes of $f(x)$ are increasing. On this interval, $f''(x)$ exists and $f''(x) > 0$. The graph of the function is above the tangent at every point on the interval.

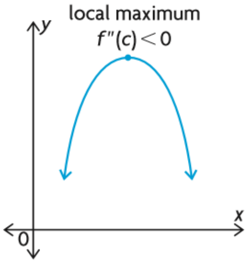
concave down The graph of $y = f(x)$ is **concave down** on an interval $a \le x \le b$ in which the slopes of $f(x)$ are decreasing. On this interval, $f''(x)$ exists and $f''(x) < 0$. The graph of the function is below the tangent at every point on the interval.

Test for concavity: If $f(x)$ is a differentiable function whose second derivative exists on an open interval *I*, then

- the graph of $f(x)$ is concave up on *I* if $f''(x) > 0$ for all values of x in *I*
- the graph of $f(x)$ is concave down on *I* if $f''(x) < 0$ for all values of x in *I*

Notice!





Second Derivative Test (for local extrema)

The second derivative test: Suppose that $f(x)$ is a function for which $f'(c) = 0$, and the second derivative of $f(x)$ exists on an interval containing c.

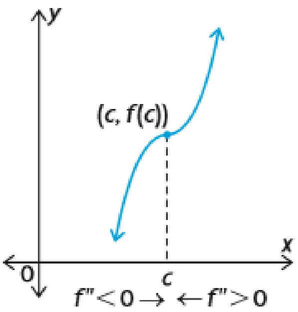
- If $f''(c) > 0$, then $f(c)$ is a local minimum value.
- If $f''(c) < 0$, then $f(c)$ is a local maximum value.
- If $f''(c) = 0$, then the test fails. Use the first derivative test.

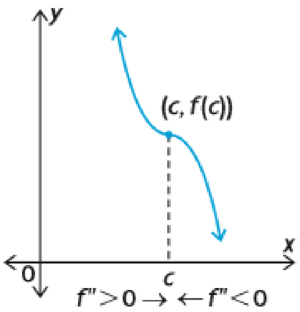
First Derivative Test

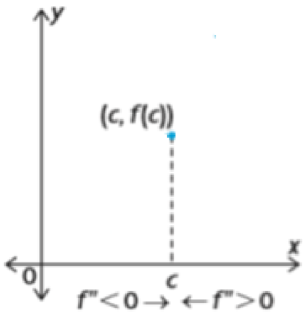
Point of Inflection

A **point of inflection** occurs at $(c, f(c))$ on the graph of $y = f(x)$ if $f''(x)$ changes sign at $x = c$. That is, the curve changes from concave down to concave up, or vice versa.

All points of inflection on the graph of $y = f(x)$ must occur either where $\frac{d^2y}{dx^2}$ equals zero or where $\frac{d^2y}{dx^2}$ is undefined.





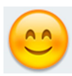


<u>Example 1</u> Apply the second derivative test to: $f(x) = x^4 - 12x^3 - 5$

<u>Example 2</u> Graph by applying the second derivative test:

$$f(x) = (x-2)^{\frac{1}{3}}$$

How are you feeling?

Concave up?  Concave down? 

Page 205 #1, 2cd, 3cd, 4c, 5b, 7, 8b, 9, 10, 11