The Curve ~~Sketching~~ Graphing Algorithm (4.5)

"Using calculus, I can graph any polynomial function, rational function and power function. I can apply what I have learned in familiar and unfamiliar settings."

B1. I can make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching;

from page 207...

An Algorithm for ~~Sketching the Graph of~~ graphing $y = f(x)$

Note: As each piece of information is obtained, use it to build the ~~sketch~~ graph.

1: Determine any discontinuities or limitations in the domain. For discontinuities, investigate the function's values on either side of the discontinuity.

2: Determine any vertical asymptotes.

3: Determine any intercepts.

4: Determine any critical ~~numbers~~ points by finding where $\frac{dy}{dx} = 0$ or where $\frac{dy}{dx}$ is undefined.

5: Determine the intervals of increase/decrease, and then test critical points to see whether they are local maxima, local minima, or neither, by using the First Derivative Test/Second Derivative Test.

6: Determine the behaviour of the function for large positive and large negative values of x. This will identify horizontal asymptotes, if they exist. Identify if the functions values approach the horizontal asymptote from above or below.

7: Determine $\frac{d^2y}{dx^2}$ and test for points of inflection using the intervals of concavity.

8: Determine any oblique asymptotes. Identify if the functions values approach the obliques asymptote from above or below.

9: Complete the ~~sketch~~ graph using the above information.

When using this algorithm, keep two things in mind:

1. You will not use all the steps in every situation. Use only the steps that are essential.

2. You are familiar with the basic shapes of many functions. Use this knowledge when possible.

Example 1 Graph $g(x) = \frac{1}{x^2+3}$

Graphing Entertainment

Graph. Show algebra for all properties.

Use desmos to verify your answer.

a) $f(x) = x^4 - 8x^3$

b) $f(x) = 3 + \frac{1}{(x+2)^2}$

c) $y = \frac{2x}{x^2 - 25}$

d) $f(x) = \frac{x^2 - 3x + 6}{x - 1}$

e) $h(x) = \frac{-2x^2 + 8x - 14}{x - 3}$

f) $f(x) = (x-4)^{\frac{2}{3}}$

g) $f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$