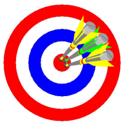
The Cross Product (7.6)

Math Learning Target:



"I can determine the cross product of any two vectors in R^3, when applicable, and I can interpret the result. I also know, and can prove, all properties of the cross product. I can apply what I have learned in familiar and unfamiliar settings."

Recall:

Section 7.4
Q# 13b

13. a. Given the vectors $\vec{p} = (-1, 3, 0)$ and $\vec{q} = (1, -5, 2)$, determine the components of a vector perpendicular to each of these vectors.
 b. Given the vectors $\vec{m} = (1, 3, -4)$ and $\vec{n} = (-1, -2, 3)$, determine the components of a vector perpendicular to each of these vectors.

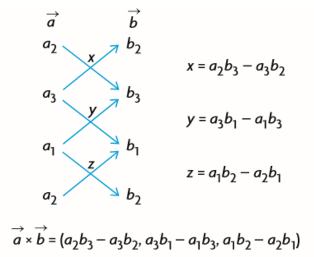
The vector $\vec{a} \times \vec{b}$ is a vector that is perpendicular to both $\vec{a}$ and $\vec{b}$, which is known as the cross product of $\vec{a}$ and $\vec{b}$, using a right-handed system. There are an infinite number of vectors that are perpendicular to these vectors, but the cross product is the one chosen in the simplest way.

Cross Product

proof is on pg. 403

The **cross product** of $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ is the vector

$$\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$$



Alternative method for calculating $\vec{a} \times \vec{b}$

Example

Using the formula for cross product, determine $\vec{m} \times \vec{n}$ for Section 7.4 Q#13b.

Example

Using the formula for cross product, determine $\vec{n} \times \vec{m}$ for Section 7.4 Q#13b.

Properties

Properties of the Cross Product

Let $\vec{p}$, $\vec{q}$, and $\vec{r}$ be three vectors in R^3, and let $k \in \mathbf{R}$.

Vector multiplication is not commutative: $\vec{p} \times \vec{q} = -(\vec{q} \times \vec{p})$,

Distributive law for vector multiplication: $\vec{p} \times (\vec{q} + \vec{r}) = \vec{p} \times \vec{q} + \vec{p} \times \vec{r}$,

Scalar law for vector multiplication: $k(\vec{p} \times \vec{q}) = (k\vec{p}) \times \vec{q} = \vec{p} \times (k\vec{q})$,

Example

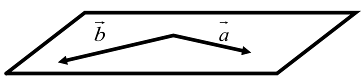
Prove the Distributive Property of cross product.

Right-handed System

The vectors $\vec{a}$, $\vec{b}$ and $\vec{a} \times \vec{b}$ always follow a right-handed system.

Example

Sketch $\vec{a} \times \vec{b}$:



MathSIP! Page 407... #1, 2, 3*, 4c, 5, 6, 7, 9a, 10, 11, 12. *in the answer for 3d, it should be $\vec{a} \cdot \vec{b}$