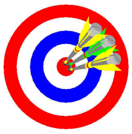
Applications of the Dot and Cross Product (7.7)

Day 1



"I can calculate the work acted upon an object. I can calculate the area of a parallelogram and triangle using vectors. I understand the proofs presented today. I can apply what I have learned in familiar and unfamiliar settings. "

Recall > The dot product is a scalar quantity. The cross product is a vector quantity.

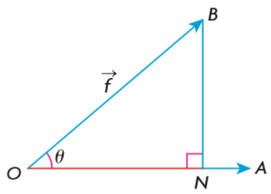
Work The energy transferred to an object by an applied force over a measured distance is the **work** acted upon the object. In other words, it is a scalar quantity equal to the *product of the distance an object has been displaced and the scalar component of the force*, along the line of displacement. It is measured in Newton-metres (Nm) or the Joule (J).

Theorem

Formula for the Calculation of Work

$W = \vec{f} \cdot \vec{s}$, where $\vec{f}$ is the force acting on an object, measured in newtons (N); $\vec{s}$ is the displacement of the object, measured in metres (m); and W is the work done, measured in joules (J).

Proof



Example Calculate the work, to the nearest joule, done by a 10 N force moving a particle from A(2, 1) to B(8, 5) when the force acts on an angle of 135 degrees to AB (metres).

Theorem For any two vectors $\vec{a}$ and $\vec{b}$: $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

where θ is the angle between the two vectors and $0^\circ \leq \theta \leq 180^\circ$

Proof

See Aside

Theorem The area of any parallelogram formed by vectors $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$

Proof

Example Outline the steps required to find the exact area of a triangle defined by points A(-3, 1, 4), B(6, 2, 0) and C(3, -1, 1).

You will love this "work"...Page 414... #2, 3ab, 4bc (read Page 412 Example 3 first), 6, 7 + START UNIT REVIEW *from the second proof on page 1:*

$$\left|\vec{a}\times\vec{b}\right|^2 = \left(a_2b_3 - a_3b_2\right)^2 + \left(a_3b_1 - a_1b_3\right)^2 + \left(a_1b_2 - a_2b_1\right)^2$$

$$= a_2^2b_3^2 - 2a_2a_3b_2b_3 + a_3^2b_2^2 + a_3^2b_1^2 - 2a_1a_3b_1b_3 + a_1^2b_3^2 + a_1^2b_2^2 - 2a_1a_2b_1b_2 + a_2^2b_1^2$$

$$+a_1^2b_1^2 + a_2^2b_2^2 + a_3^2b_3^2 - a_1^2b_1^2 - a_2^2b_2^2 - a_3^2b_3^2$$

$$= a_1^2b_1^2 + a_1^2b_2^2 + a_1^2b_3^2 + a_2^2b_1^2 + a_2^2b_2^2 + a_2^2b_3^2 + a_3^2b_1^2 + a_3^2b_2^2 + a_3^2b_3^2$$

$$-\left(a_1^2b_1^2 + a_2^2b_2^2 + a_3^2b_3^2 + 2a_1a_2b_1b_2 + 2a_1a_3b_1b_3 + 2a_2a_3b_2b_3\right)$$

$$= \left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right) - \underbrace{\left(a_1b_1 + a_2b_2 + a_3b_3\right)}^2$$

because...

$$\left(a+b+c\right)^2$$

$$= \left(a+b+c\right)\left(a+b+c\right)$$

$$= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

Return