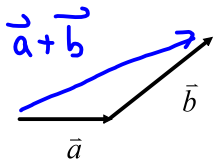
Multiplication of a Vector by a Scalar (6.3)

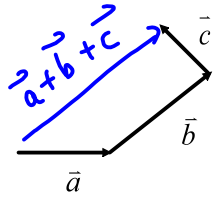
Math Learning Target:

"I understand the geometric significance of the unit vector, zero vector and multiplying a vector by a scalar. I can determine if vectors considered are collinear or not. I can solve problems involving the unit and zero vectors, and vectors multiplied by a scalar. I can apply what I have learned in familiar and unfamiliar settings."

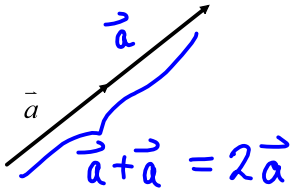
Recall: **Scalar, Vector, Equal Vector, Opposite Vector**

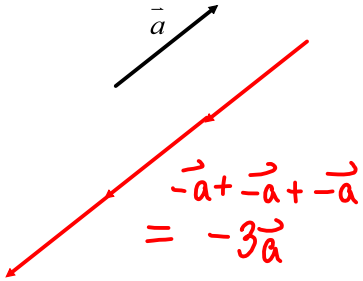
Vector Addition





What is the effect of multiplying a vector $\vec{a}$ by a scalar k ?





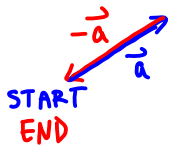
Theorem For the vector $k\vec{a}$, where k is a scalar and $\vec{a}$ is a non-zero vector:

- if $k>1$, $k\vec{a}$ is a vector with a larger magnitude than $\vec{a}$, and in the same direction.
- if $0<k<1$ ... smaller magnitude ... same direction.
- if $-1<k<0$ ... smaller magnitude ... opposite direction.
- if $k<-1$ .... larger magnitude ... opposite direction.

Zero Vector The **zero vector** is a vector of magnitude 0, with an undefined direction. We write $\vec{0}$

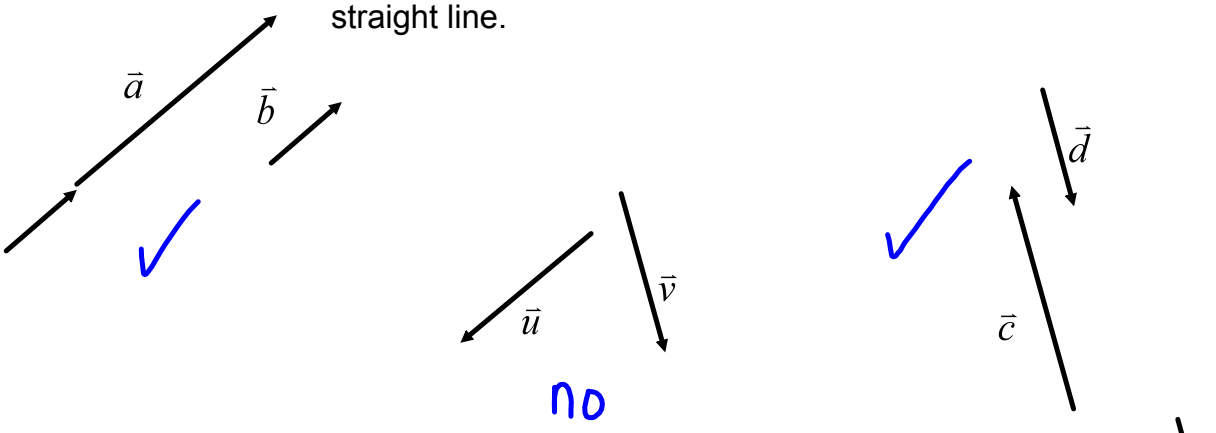
There are several ways to create it, such as:

$\vec{a}+(-\vec{a})=\vec{0}$



Collinearity

Any two vectors are **collinear** when they are parallel or lie on the same straight line. They can be described as collinear because vectors are allowed to be translated to lie on the same straight line.



no

Collinearity Theorem

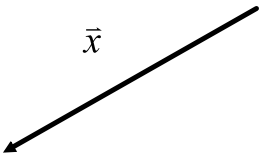
Given: two non-zero vectors $\vec{a}$ and $\vec{b}$.

$\vec{a}$ and $\vec{b}$ are collinear if and only if $\vec{a} = k\vec{b}$, for some non-zero $k \in \mathbb{R}$.

Unit Vector
(loosely stated)

a vector with magnitude = 1.

Example Given $\vec{x}$ where $|\vec{x}| = 4$, interpret $\frac{1}{4}\vec{x}$.



$\left|\frac{1}{4}\vec{x}\right| = \left|\frac{1}{4}\right||\vec{x}| = \frac{1}{4}|\vec{x}| = \frac{1}{4}(4) = 1$

Now, $\frac{1}{4}\vec{x}$ is in the same direction as $\vec{x}$. Thus $\frac{1}{4}\vec{x}$ is a unit vector corresponding to $\vec{x}$.

Unit Vector
(formally stated)

A unit vector corresponding to $\vec{x}$ is $\frac{1}{|\vec{x}|}\vec{x}$. That is, a vector of magnitude 1, and in the same or opp. direction to $\vec{x}$.

Rationalize all denominators. Do not round any scalar values and magnitudes.

Entertainment: Page 298...#1, 2bd, 6e, 7a*, 10, 13, 15, 17, 20, 21
*answer in back of text is partially incorrect