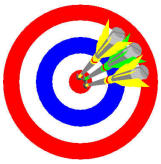
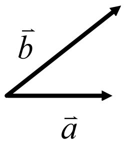
Properties of Vectors (6.4)

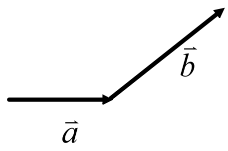
Math Learning Target:



"I can prove and apply the commutative, associative and distributive properties of vector addition. I can apply other properties, including the associative and distributive laws of scalar multiplication. I can apply what I have learned in familiar and unfamiliar settings."

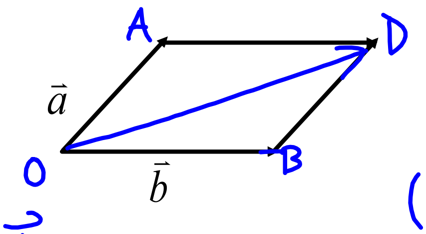
Recall: **Vector Addition**





Commutative Property of Vector Addition

$$\vec{a}+\vec{b}=\vec{b}+\vec{a}$$



$\overrightarrow{OD} = \vec{a}+\vec{b}$

$\overrightarrow{OD} = \overrightarrow{OA}+\overrightarrow{OB}$

(by Parallelogram Law)

(By the Triangle Law)

In ΔOBD,

$\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{BD}$

$\overrightarrow{OD}=\vec{b}+\vec{a}$

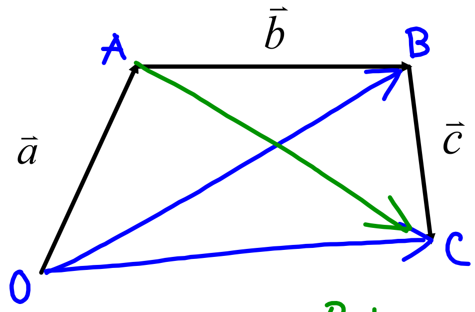
Thus,

$\overrightarrow{OD}=\overrightarrow{OD}$

$\vec{a}+\vec{b}=\vec{b}+\vec{a}$ $\square$

Associative Property of Vector Addition

$$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$$



$\overrightarrow{OB}=(\vec{a}+\vec{b})$

$\overrightarrow{OC}=(\vec{a}+\vec{b})+\vec{c}$

and

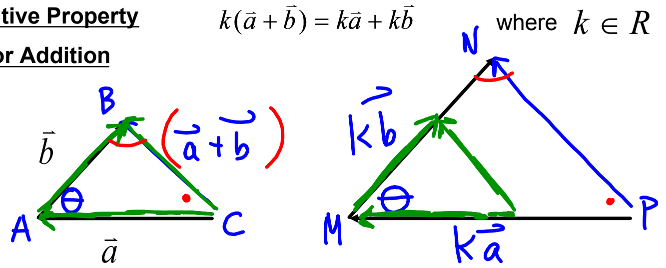
$\overrightarrow{AC}=(\vec{b}+\vec{c})$

$\overrightarrow{OC}=\vec{a}+(\vec{b}+\vec{c})$

But $\overrightarrow{OC}=\overrightarrow{OC}$

Thus, $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ $\square$

Distributive Property of Vector Addition $k(\vec{a}+\vec{b}) = k\vec{a} + k\vec{b}$ where $k \in R$



For $k>0$, create $k\vec{a}, k\vec{b}$:

$\Delta ABC \sim \Delta MNP$ since all corresponding angles are equal. Thus, all corresponding magnitudes are proportional (i.e. scale factor is k)

Thus, $\vec{PN} = k\vec{CB}$

OR $\vec{PN} = k(\vec{a}+\vec{b})$

By the Triangle Law, $\vec{PN} = k\vec{a} + k\vec{b}$

Now, $\vec{PN} = \vec{PN}$

$\therefore k(\vec{a}+\vec{b}) = k\vec{a} + k\vec{b}$ $\square$

Further Properties

If $\vec{a}$ is a vector, and m and n are real numbers:

1. Each nonzero vector $\vec{a}$ has a corresponding negative vector $-\vec{a}$ such that $\vec{a} + (-\vec{a}) = \vec{0}$
2. $1\vec{a} = \vec{a}$
3. $\vec{a} + \vec{0} = \vec{a}$
4. $m(n\vec{a}) = (mn)\vec{a} = mn\vec{a}$
5. $(m+n)\vec{a} = m\vec{a} + n\vec{a}$

Example If $\vec{a} = 3\vec{x} - 5\vec{y}$ and $\vec{b} = -2\vec{x} + 9\vec{y}$, find $5\vec{a} - 9\vec{b}$

$= 5(3\vec{x} - 5\vec{y}) - 9(-2\vec{x} + 9\vec{y})$

$= 15\vec{x} - 25\vec{y} + 18\vec{x} - 81\vec{y}$ distributive vector add.

$= 15\vec{x} + 18\vec{x} - 25\vec{y} - 81\vec{y}$ commutative vector add.

$= (15+18)\vec{x} + (-25-81)\vec{y}$ distributive vector add.

$= 33\vec{x} - 106\vec{y}$

MathSIP: Page 306 #3, 6, 7, 8, 9, 12, 4*

*this is challenging; the answer in the back does not demonstrate the proof