

Expand and simplify $(a+b)^6$

$6C_1$

$$= 6$$

OR $\binom{6}{1}$

$$= 6$$

Row 0:					1										
Row 1:					1	1									
Row 2:					1	2	1								
Row 3:					1	3	3	1							
Row 4:					1	4	6	4	1						
Row 5:					1	5	10	10	5	1					
Row 6:					1	6	15	20	15	6	1				
Row 7:					1	7	21	35	35	21	7	1			
Row 8:					1	8	28	56	70	56	28	8	1		
Row 9:					1	9	36	84	126	126	84	36	9	1	
Row 10:					1	10	45	120	210	252	210	120	45	10	1

Pascal's
Triangle

$$\boxed{{}^nC_r}$$

$$(a+b)^6 = 1a^6b^0 + 6a^5b^1 + 15a^4b^2 + 20a^3b^3$$

$$+ 15a^2b^4 + 6ab^5 + 1a^0b^6$$

$$= \binom{6}{0}a^6b^0 + \binom{6}{1}a^5b^1 + \binom{6}{2}a^4b^2 + \binom{6}{3}a^3b^3$$

$$+ \binom{6}{4}a^2b^4 + \binom{6}{5}ab^5 + \binom{6}{6}a^0b^6$$

$$\binom{n}{0} = 1, n \in \mathbb{N}$$

$$\binom{n}{n} = 1, n \in \mathbb{N}$$

If $f(x) = x^n$, prove $f'(x) = nx^{n-1}$, $n \in \mathbb{N}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \text{ if limit exists.}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\binom{n}{0}x^n h^0 + \binom{n}{1}x^{n-1}h^1 + \binom{n}{2}x^{n-2}h^2 + \dots + \binom{n}{n-1}x^1 h^{n-1} + \binom{n}{n}x^0 h^n - x^n}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{x^n} + \binom{n}{1}x^{n-1}h^1 + \binom{n}{2}x^{n-2}h^2 + \dots + \binom{n}{n-1}x^1 h^{n-1} + \binom{n}{n}x^0 h^n - \cancel{x^n}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h} x^{n-1} \left[\binom{n}{1} + \binom{n}{2}h x^{-1} + \dots + \binom{n}{n-1}x^{-n+2} h^{n-2} + \binom{n}{n}x^{-n+1} h^1 \right]}{h}$$

$$= \lim_{h \rightarrow 0} x^{n-1} \left[\binom{n}{1} + 0 + 0 + \dots + 0 + 0 \right]$$

$$= \binom{n}{1} x^{n-1}$$

$$= n x^{n-1} \quad \square$$