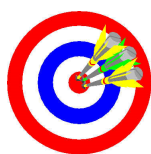


Determining Average Rate of Change (2.1)



Math Learning Target:

"I can calculate an average rate of change, and I can interpret the result. I can apply what I have learned in familiar and unfamiliar settings."

rate

A **rate** is a comparison of one quantity to another, measured in different units.

rate of change

(informally stated)

A **rate of change** is a change in one quantity relative to the change in another quantity, measured in different units.

average rate of change

Given a relation R , an **average rate of change** is a change in the quantity of the dependent variable (y) relative to the change in the quantity of the independent variable (x), on the interval $\{x \in \mathbb{R} \mid x_1 \leq x \leq x_2, x_1 \neq x_2\}$

If the relation is a function $y = f(x)$ then the average rate of change is calculated by:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Notice! ... it represents the slope connecting $(x_1, f(x_1))$ and $(x_2, f(x_2))$.

secant line

A **secant line** is a line drawn on a graph of a relation that connects two points.

EXAMPLE

A rocket is shot vertically off a cliff. The height of the rocket, in m , is given by $s(t) = -5t^2 + 60t + 220$ where t is in seconds.

- Find the average rate of change on $\{t \in \mathbb{R} \mid 12 \leq t \leq 14\}$. Interpret.
- Find the average rate of change on $t \in [4, 8]$. Interpret.

EXAMPLE

State any function with a:

a) constant average rate of change.

b) variable average rate of change.

$$f(x) = x$$

$$f(x) = \sin x$$

$$\begin{aligned}
 \text{a) } \text{aROC} &= \frac{s(t_2) - s(t_1)}{t_2 - t_1} \\
 &= \frac{[-5(14)^2 + 60(14) + 220] - [-5(12)^2 + 60(12) + 220]}{14 - 12} \\
 &= \dots \\
 &= -70
 \end{aligned}$$

The toy rocket is decreasing its height by 70 m/s, **on average**, from 12 to 14 seconds.

$$\begin{aligned}
 \text{b) } \text{aROC} &= \frac{\Delta s}{\Delta t} \\
 &= \frac{[-5(8)^2 + 60(8) + 220] - [-5(4)^2 + 60(4) + 220]}{8 - 4} \\
 &= \dots \\
 &= 0
 \end{aligned}$$

Hence, **on average**, from 4 to 8 seconds, its height above the

ground has not changed