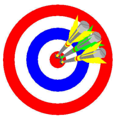
2.5 Solving Problems involving Rates of Change

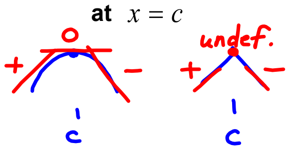


"I know...

- what global (absolute) maximums and minimums, and I can identify them on a graph;
- what local maximums and minimums, and I can identify them on a graph;
- how use the algebraically simplified difference quotient to determine the presence of a local minimum and local maximum.

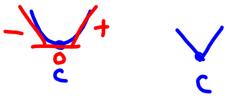
Recall: The calculations for rate of change at $x = a$ and slope of the tangent at $x = a$ are identical.

Local Maximum at $x = c$



Given $y = f(x)$. For values of x near c ... A **local maximum** at $x = c$ exists if the function's rate of change changes from positive to negative through $x = c$. At $x = c$ the rate of change will be zero (or undefined).

Local Minimum at $x = c$



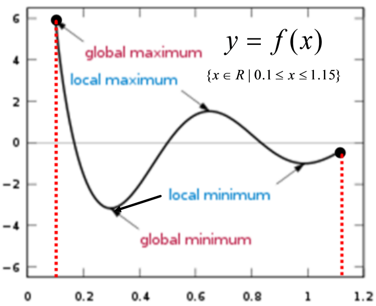
Given $y = f(x)$. For values of x near c ... A **local minimum** at $x = c$ exists if the function's rate of change changes from negative to positive through $x = c$. At $x = c$ the rate of change will be zero (or undefined).

Global Maximum at $x = c$

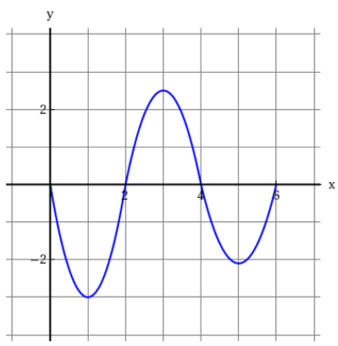
Given $y = f(x)$. A **global (absolute) maximum** at $x = c$ exists if $f(c) > f(x)$ for all values of x in the function's domain.

Global Minimum at $x = c$

Given $y = f(x)$. A **global (absolute) minimum** at $x = c$ exists if $f(c) < f(x)$ for all values of x in the function's domain.



Given a function $y = f(x)$ defined on $0 \le x \le 6$:



For this function's domain, state the integer(s) x that correspond to a…

Local minimum	Local maximum	Global minimum	Global maximum
$x = 1$ and $x = 5$	$x = 3$	$x = 1$	$x = 3$

Recall:

average rate of change

Given: $y = f(x)$. The average rate of change is calculated by:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad \div \quad \textbf{difference quotient}$$

When one uses the average rate of change (difference quotient) carefully, one can estimate the rate of change.

Example Using the *algebraically simplified* difference quotient, prove that a local minimum value occurs at $x = 3$ for $f(x) = x^2 - 6x + 5$

USE THE *ALGEBRAICALLY SIMPLIFIED DIFFERENCE QUOTIENT* FOR ALL RATE OF CHANGE CALCULATIONS:

Page 111 #1, 3, 4, 6cd, 9a, 10, 14

Careful! x is not 1500

$$m_{tangent} = \frac{C(1.5+h) - C(1.5)}{(1.5+h) - 1.5}$$

$$= \frac{[0.3(1.5+h)^2 - 0.9(1.5+h) + 1.675] - [0.3(1.5)^2 - 0.9(1.5) + 1.675]}{h}$$

$$= \frac{[0.3(1.5+h)(1.5+h) - 1.35 - 0.9h + 1.675] - [\ 1\ ]}{h}$$

$$= \frac{0.3(2.25 + 3h + h^2) + 0.325 - 0.9h - 1}{h}$$

$$= \frac{0.675 + 0.9h + 0.3h^2 - 0.9h - 0.675}{h}$$

$$= \frac{0.3h^2}{h}$$

$=$ Keep going

Example

Using the *algebraically simplified* difference quotient, prove that a local minimum value occurs at $x = 3$ for $f(x) = x^2 - 6x + 5$

gr 10, 11 : complete the square
OR
factor/axis of symmetry

gr 12 following $3 \le x \le 3+h$ i.e. $h = 0.001$
OR preceeding $3+h \le x \le 3$ i.e. $h = -0.001$

$$\text{iroc} \doteq \text{aroc} = \frac{f(3+h) - f(3)}{3+h-3}$$

$$= \frac{[(3+h)^2 - 6(3+h) + 5] - [(3)^2 - 6(3) + 5]}{h}$$

$$= \frac{(3+h)(3+h) - 18 - 6h + 5 - [-4]}{h}$$

$$= \frac{9 + 3h + 3h + h^2 - 18 + 5 + 4 - 6h}{h}$$

$$= \frac{h^2}{h}$$

$m_{tangent} = h$

preceding $h \doteq -0.001$ — following $h \doteq 0.001$

iroc $= 0$ — iroc $= 0$

	$x = 2.999$	$x = 3$	$x = 3.001$
$m_{tangent}$	$-$	0	$+$

Hence, a local minimum at $x = 3$ $\square$