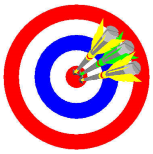
Factoring Polynomials: *Part 1*



"I can state the Remainder Theorem and the Factor Theorem. I can always apply them, when applicable. I can apply what I have learned in familiar and unfamiliar settings."

Activity

In a group, using small response boards:

Given: $p(x) = x^3 + 2x^2 - 5x - 6$

1. Evaluate: $p(-2)$ and $p(1)$
2. Divide $p(x)$ by $x + 2$
3. Divide $p(x)$ by $x - 1$
4. Describe what you have learned.

The Remainder Theorem

Given polynomial $p(x)$ and $a \in \mathbb{R}$, then $p(a)$ is the remainder if $p(x)$ is divided by $x - a$.

***Recall:* The Division Statement**

Dividend = Divisor × Quotient + Remainder

The Factor Theorem

Given polynomial $p(x)$ and $a \in \mathbb{R}$: $x - a$ is a factor of $p(x)$ if and only if $p(a) = 0$

Example Factor fully (over the rationals), if possible:

Let $p(x) = x^3 - 6x^2 - x + 30$

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$\because p(3)=0 \quad \therefore x-3$ is a factor

Let $q(x)$ be the quotient

$$p(x)=(x-3)\,q(x)$$

$$
\begin{array}{r|l}
 & x^2-3x-10 \\
\hline
x-3 & x^3-6x^2-x+30 \\
 & \underline{x^3-3x^2} \\
 & \quad -3x^2-x \\
 & \quad \underline{-3x^2+9x} \\
 & \qquad -10x+30 \\
 & \qquad \underline{-10x+30} \\
 & \qquad\qquad 0
\end{array}
$$

$$\therefore p(x)=(x-3)(x^2-3x-10)$$

$$p(x)=(x-3)(x+2)(x-5)$$

$\square$