

3.7 Factoring a Sum and Difference of Cubes



"I can factor fully any polynomial, including a Sum and Difference of Cubes. I can apply what I have learned in familiar and unfamiliar settings."

Sum of Cubes:

$$y^3 + 2^3$$

Difference of Cubes:

$$2x^3 - 3^3$$

Apply the Factor Theorem to factor completely:

$$B^2 - 4AC < 0$$

a) $y^3 + 8$

$$= (y+2)(y^2 - 2y + 4)$$

$$B^2 - 4AC = (-2)^2 - 4(1)(4) < 0$$

b) $8x^3 - 27$

$$= (x-1.5)(8x^2 + 12x + 18)$$

$$= 2(x-1.5)(4x^2 + 6x + 9)$$

$$= (2x-3)(4x^2 + 6x + 9)$$

Predict a formula for a factored Difference of Cubes: $a^3 - b^3$

$$= (a-b)(a^2 + ab + b^2)$$

Predict a formula for a factored Sum of Cubes: $a^3 + b^3$

$$= (a+b)(a^2 - ab + b^2)$$

The Factor Theorem can be applied to any expression. However, it may be more faster to use one of the new formulae if one recognizes the expression as a sum/difference of cubes. Hence, the following algorithm is suggested, from now on, when required to factor:



Is the expression a sum/difference of cubes?
If so, use the appropriate formula. Otherwise, apply the Factor Theorem directly.

MathSIP! pg 182 #2acegi, 3, 4acegi, 5ac, 6
Are you factoring fully?

18a) Consider y and y^2 to be constants, etc.

$$\begin{array}{r}
 x^2 \\
 \hline
 x^2 + 0x + y^2 \) \ x^4 + yx^3 + 0y^2x^2 - y^3x - y^4 \\
 \underline{x^4 + 0yx^3 - y^2x^2} \quad \downarrow \\
 yx^3 + y^2x^2 - y^3x
 \end{array}$$

etc. . .

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#9) $p(x) = 12x^3 + kx^2 - x - 6$

if $2x - 1$ is a factor
then so are $2(x - \frac{1}{2})$!!!

$$x - a$$

$$p\left(\frac{1}{2}\right) = \quad = 0$$