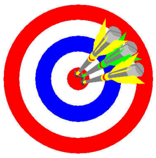
Higher Order Derivatives. Velocity. Acceleration. (3.1)



"Given an object's position function, I can find its position, velocity and acceleration at any point in time, and interpret its motion at this time. I can apply what I have learned in familiar and unfamiliar settings."

A1. "I can demonstrate an understanding of rate of change by making connections between average rate of change over an interval and instantaneous rate of change at a point, using the slopes of secants and tangents and the concept of the limit";

B1. "I can make connections, graphically and algebra... the key features of a functio... d second derivatives, ...nnections in curve sketc...

B2. "I can s... ncluding optimization problems, that require the use of the concepts and procedures associated with the derivative, including problems arising from real-world applications and involving the development of mathematical models."

Recall:
(Chpt. 1)

Displacement: the change in position of an object, in a given direction.

Velocity: the rate of change in position of an object, per unit of time, in a given direction. Thus, it is the rate of displacement per unit of time, in a given direction.

Acceleration: the rate of change of velocity, per unit of time, in a given direction.

All of the above are vector quantities.

COMING SOON!...

The velocity of an object is relative to a frame of reference.

This frame of reference influences the stated velocity of the object.

Second Derivative

The derivative of the derivative is the **second derivative,** if it exists. The second derivative is also a function.

$y=f(x)$ $\qquad$ $y'=\frac{dy}{dx}$

$y'=f'(x)$

$y''=f''(x)$ $\qquad$ $y''=\frac{d^2y}{dx^2}$

We will always assume that motion is in a straight line.

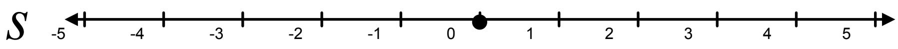
When studying linear motion, we will think of the object moving along a real number line, which gives us an origin of reference, as well as positive and negative directions.

The position of an object along a line, relative to the origin, at time t is denoted by the function $s(t)$. <u>This is known as the position or displacement function</u>. The value of $s(t)$ is positive when it is right (up) from the origin, and negative when the object is left (down) relative to the origin.

The rate of change of $s(t)$ with respect to time is: $v(t)=s'(t)$

The rate of change of $v(t)$ with respect to time is: $a(t)=v'(t)=s''(t)$

Since the object's motion is along the real number line, it makes sense that when the object is moving right at time t that $v(t) > 0$. When the object moves left at time t that $v(t) < 0$. If $v(t) = 0$ the object is at rest.



The object is accelerating if: (speeding up)

$v(t) > 0$ and $a(t) > 0$, or

$v(t) < 0$ and $a(t) < 0$.

MEMORIZE

The object is decelerating if: (slowing down)

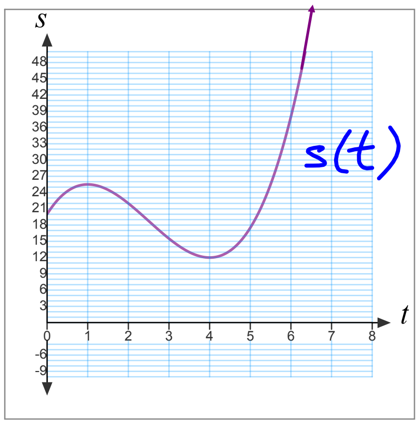
$v(t) > 0$ and $a(t) < 0$, or

$v(t) < 0$ and $a(t) > 0$.

Example The position of an object at t seconds, moving s metres in a straight line is modelled by:

$$s(t) = t^3 - 7.5t^2 + 12t + 20$$

Algebraically for a) to d):



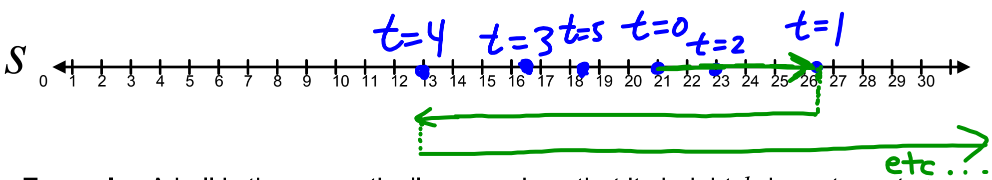
a) Determine the initial position, velocity and acceleration. Discuss the object's motion.

b) Repeat part a) for $t = 2$ seconds.

c) When is the object moving down (left)?

d) When does the object turn directions?

e) Construct a position diagram below every second, starting at zero seconds.



Example A ball is thrown vertically upward, so that its height h in metres at t seconds is: $h(t) = 15t - 5t^2$ displacement

a) What is the initial velocity?
b) Find the maximum height reached by the ball.
c) Determine the velocity of the ball as it lands on the ground.

Don't be an object at rest :)

Page 127... #2*eg, 3e*****f, 4**, 5***, 8, 9, 10****, 12, 13, 14, 16.
*simplified/factored form!
**answer for 4b ii) is 7 < t
***answer for 5c should be t = 3
****answer for 10a should be 105/4 and 10d should be 0 < t < 3 secs
*****3e answer in the back should have negative numbers in the exponents

$$s = t^3 - 7.5t^2 + 12t + 20$$

a) Let $t = 0$

$s(0) = \ldots = 20$ m

$v(t) = 3t^2 - 15t + 12$

$v(0) = \ldots = 12$ m/s

$a(t) = 6t - 15$

$a(0) = \ldots = -15$ m/s/s

Object is initially 20 m, from the origin (0). It is moving to the right (up), and it is slowing down.

b) Let $t = 2$

$s(2) = \ldots = 22$ m

$v(2) = \ldots = -6$ m/s

$a(2) = -3$ m/s^2.

The object is 22 m from the origin (0). It is moving to the left (down) since $v(2)$ is negative. It is speeding up since both $v(2)$ and $a(2)$ are negative.

c) Solve $v(t) < 0$

$$3t^2 - 15t + 12 < 0$$
$$t^2 - 5t + 4 < 0$$
$$(t-4)(t-1) < 0$$

Let $f(t) = (t-4)(t-1)$

zeros: $\{t \in \mathbb{R} \mid t = 4\} \cup \{t \in \mathbb{R} \mid t = 1\}$

intervals of t	$0 \le t < 1$	$1 < t < 4$	$t > 4$
sign of $f(t)$	+	−	+

Hence, travelling left on

$\{t \in \mathbb{R} \mid 1 < t < 4\}$ □

d) Solve $v(t) = 0$

$$3t^2 - 15t + 12 = 0$$
$$t^2 - 5t + 4 = 0$$
$$(t-4)(t-1) = 0$$

Change direction at $\{t \in \mathbb{R} \mid t = 1\} \cup$

$\{t \in \mathbb{R} \mid t = 4\}$ ☑

$h(t) = 15t - 5t^2$

a) $v(t) = h'(t) = 15 - 10t$

Let $t = 0$

$v(0) = \dots = 15$ m/s initial velocity

b) Let $v(t) = 0$

$0 = 15 - 10t$

$t = 1.5$

sub in $t = 1.5$ into $h(t)$:

$h(1.5) = \dots = 11.25$ m

max height

c) Let $h(t) = 0$

$0 = 15t - 5t^2$

$0 = 3t - t^2$

$0 = t(3 - t)$.

$\{t \in R \mid t = 0\} \cup \{t \in R \mid t = 3\}$

inadmissible (time value at launch)

So $v(3) = \dots = -15$ m/s

Hence 15 m/s [down]

□