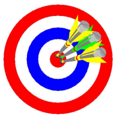
Vectors as Velocities (7.2)



"I understand why velocity is stated relative to a frame of reference. I can calculate any velocity, displacement, and time value, in a given problem. I can apply what I have learned in familiar and unfamiliar settings."

Recall: **Velocity** is the rate of change in position of an object, per unit of time, in a given direction. In other words, it is the rate of displacement per unit of time, in a given direction.

For simplicity, in the vector~~s~~ section only, we assume $\vec{v} = \vec{v}_{avg}$.

$$\vec{v}_{avg} = \frac{\Delta d}{\Delta t}$$

$$\Delta \vec{d} = \Delta t \, \vec{v}_{avg}$$

All motion is relative to a frame of reference.

The velocity of an object is stated relative to a frame of reference. This frame of reference influences the stated velocity of the object. The frame of reference can be thought of as a state of motion of an "observer".

Air Velocity The velocity of an object in the air, such as a plane, relative to a fixed point in the air. It does not include the effect of the wind.

Water Velocity The velocity of an object in the water, such as a boat, relative to a fixed point in the water. It does not include the effect of the current.

Ground Velocity The velocity of an object relative to a fixed point on the ground. It includes the effect of the wind or current, if applicable.

Example 1
One-dimensional motion.

Suppose a large cruise ship is moving at 5 m/s [S], relative to the shore. A passenger, out her for daily jog, is jogging at 1 m/s [S], relative to the boat. Relative to the shore, what is her velocity?

Example 2
Two-dimensional motion.

Suppose the passenger in the previous example is now jogging at 1 m/s [E], relative to the boat, as the boat travels of 5 m/s [S] relative to the shore. Determine her ground velocity. Round all values to the nearest tenth.

Example 3

A river is 900 m wide and flows at 2 m/s. Paula paddles a canoe at a speed of 4 m/s in still water. A dock and a marina are directly across from each other on the river. She paddles perpendicular to the current.

a) How far downstream from the marina on the shore will she land?

b) How long does it take for her to cross the river?

c) New scenario. If Paula decides she wants to land at the marina, at what direction must she head, to the nearest degree? What is her resultant ground velocity? What is her water velocity?

Your "current" questions: Page 369 # 2, 4*, 5, 7, 10**, 11, 12, 13***, 14

* the answer is 30 degrees upstream or 60 degrees relative to the shore upstream
**the direction is not given in the Answers section. It is about 53 degrees downstream OR 37 degrees relative to the shore downstream.
***the answer for 13a should be about 66m (NOT 68m!), and for 13b it should be 66 2/3 seconds

Example 1

One-dimensional motion.

↑N

Suppose a large cruise ship is moving at 5 m/s [S], relative to the shore. A passenger, out her for daily jog, is jogging at 1 m/s [S], relative to the boat. Relative to the shore, what is her velocity? ground velocity

Let $\vec{c}$ be vector rep. cruise ship.
Let $\vec{p}$ " " " passenger.
Let $\vec{v}$ be the resultant ground velocity.

•

$\downarrow \vec{c}$
$\downarrow \vec{p}$

$$\vec{v} = \vec{c} + \vec{p}$$

and $|\vec{v}| = |\vec{c}| + |\vec{p}|$
$= 5 + 1$
$= 6$

Thus, 6 m/s, [S] $\square$

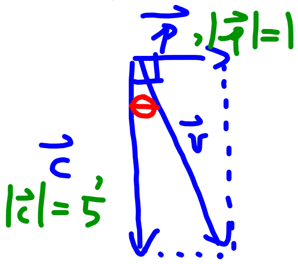
Example 2

Two-dimensional motion.

Suppose the passenger in the previous example is now jogging at 1 m/s [E], relative to the boat, as the boat travels of 5 m/s [S] relative to the shore. Determine her ground velocity. Round all values to the nearest tenth.

↑N

(same "let" statements as Ex 1)



$|\vec{v}|^2 = (1)^2 + (5)^2$

$|\vec{v}| = \sqrt{26}$

$|\vec{v}| \doteq 5.1$

Let θ be angle of her relative to direction of boat.

$\tan\theta = \frac{1}{5}$

$\theta \doteq 11.3^\circ$

Hence, her resultant ground velocity is about 5.1 m/s, [S11.3°E]

□

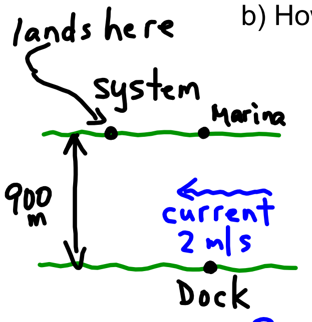
Example 3

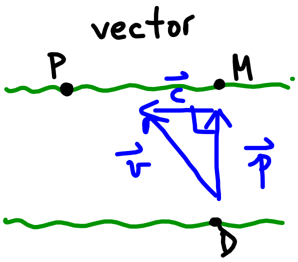
A river is 900 m wide and flows at 2 m/s. Paula paddles a canoe from a dock at a speed of 4 m/s in still water. A dock and a marina are directly across from each other on the river. She paddles perpendicular to the current.

Let $\vec{p}$ be vector rep. Paula in canoe.
Let $\vec{c}$ be " " current.
Let $\vec{v}$ be her resultant velocity

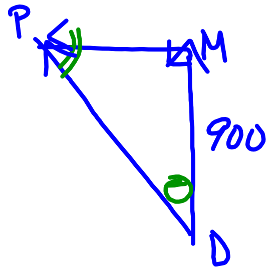
a) How far downstream from the marina on the shore will she land?

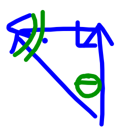
b) How long does it take for her to cross the river?





similar triangles since all corresponding angles are equal.





Let scale factor be k.

Recall:

$$\Delta\vec{d} = \Delta t\,\vec{v}_{avg}$$

Find $|\vec{v}|$ (same as $|\vec{v}_{avg}|$)

$$|\vec{v}|^2 = (4)^2 + (2)^2$$
$$|\vec{v}| = 2\sqrt{5}$$

Now, $k = \frac{900}{4} = 225$

Thus, $\Delta\vec{d} = (225)(2) = 450\text{ m}$ □

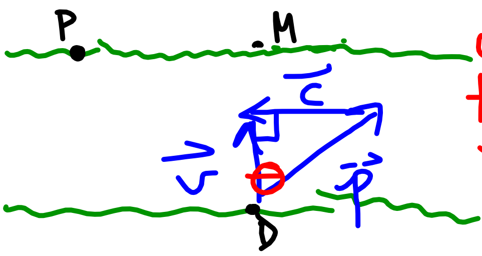
b) $\Delta t = \frac{|\vec{d}|}{|\vec{v}|}$

$$\Delta t = \frac{450}{2\sqrt{5}} = \frac{225}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{225\sqrt{5}}{5}$$

$$= 45\sqrt{5}\text{ seconds}$$ □

c) New scenario. If Paula decides she wants to land at the marina, at what direction must she head, to the nearest degree? What is her resultant ground velocity? What is her water velocity?

vector



Let θ be angle upstream relative to her ground velocity.

$$|\vec{v}|^2 = |\vec{p}|^2 - |\vec{c}|^2$$

$$|\vec{v}| = \sqrt{12} = 2\sqrt{3} \text{ m/s}$$

$$\sin\theta = \frac{2}{4} = \frac{1}{2}$$

$$\theta = 30^\circ$$

Thus, $2\sqrt{3}$ m/s, [90° rel. to shore] is her ground velocity.

Her water velocity is 4 m/s, [60° relative to shore]

OR

30° relative to her ground velocity.